Int J Theor Phys (2007) 46: 2923–2927
DOI 10.1007/s10773-007-9404-4

Hawking Radiation as Tunneling from the Vaidya–Bonner Black Hole

ShuZheng Yang · DeYou Chen

Published online: 30 May 2007

Abstract Applying the Hamilton–Jacobi method, we investigate the Hawking radiation as tunneling from the non-stationary Vaidya–Bonner black hole by considering the unfixed background space-time and self-gravitational interaction. The result shows the actual radiation spectrum deviates from the purely thermal one and the tunneling rate is related not only to the change of Bekenstein–Hawking entropy but also to the integral to the black hole mass and charge. This implies information loss is possible.

Keywords Vaidya–Bonner black hole · Self-gravitation · Radiation spectrum · Hamilton–Jacobi method

At the beginning of 1970s, Stephen Hawking put forward and proved the existence of the thermal radiation of black holes [5, 6], which plays a great role on the further cognition and research on black holes. Since the background space-time is treated as fixed and energy conservation, charge conservation and angular momentum conservation weren't taken into account, the derived Hawking radiation spectrums in the subsequent researches are purely thermal [4, 12, 18, 21]. The Hawking radiation is factually a quantum tunneling procedure, the derived spectrum in the really tunneling procedure should be not purely thermal and its tunneling rate is related with the change of Bekenstein–Hawking entropy. This is mentioned in Refs. [15–17], where the energy conservation and the self-gravitation interaction were taken into account. Besides, a significant technique is to introduce the Painlevé coordinate transformation so as to eliminate the coordinate singularity. Then it becomes relatively rather convenient to research the Hawking tunneling radiation of black holes. According to their method, much efforts has been made to the Hawking tunneling radiation in static and stationary cases, such as Hemming and Keski-Vakkuri to the anti-de Sitter background

S. Yang (✉) · D. Chen
Institute of Theoretical Physics, China West Normal University, Nanchong, Sichuan 637002, China
e-mail: szyang_sc@sohu.com

D. Chen
e-mail: deyouchen@126.com

space-time [8], Medved to the de Sitter background space-time [13] and Zhao, Zhang and Yang to the stationary axisymmetric space-times [19, 23]. In 2005, Zhang and Zhao extended the method further to the particle with mass and charge [23–26]. However, these researches focus on discussing the Hawking tunneling radiation of static and stationary black holes [3, 11, 20], while the non-stationary case has been still without deep researched. It's well known that, the absorption and evaporation of the black hole causes that its parameters as mass etc. aren't constant instead of changing with time. Then, generally speaking, black holes should be non-stationary and to research the Hawking tunneling radiation of the black hole is rather meaningful.

In this paper, the Hawking radiation as tunneling from the non-stationary and spherically symmetric Vaidya–Bonner black hole is studied by the Hamilton–Jacobi method. Firstly the action of the radiation particle is obtained according to the exact solution method. Then considering the self-gravitation interaction as well as conservation of energy and charge to study its action, we use WKB approximation to obtain the particle's tunneling rate. The result shows its radiation spectrum isn't purely thermal and the tunneling rate is related not only to the change of Bekenstein–Hawking entropy but also to the integral to the black hole mass and charge.

The line element of the non-stationary Vaidya–Bonner black hole [2] represented in advanced Eddington coordinates is given by

$$ds^2 = -\left(1 - \frac{2M(v)}{r} + \frac{Q^2(v)}{r^2}\right)dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

with the electromagnetic potential $A_\mu = (\frac{Q(v)}{r}, 0, 0, 0)$, where $M(v)$ and $Q(v)$ represent the mass and charge of the black hole changes with time respectively. When $Q(v) = 0$, the non-stationary Vaidya–Bonner black hole is reduced to the Vaidya black hole. The line element is different from that of the stationary sphere symmetrically charged black hole and the coordinate singular isn't existed. The event horizon and entropy of the black hole are respectively

$$r_h = \frac{M(v) + \sqrt{M^2(v) - Q^2(v)(1 - 2\dot{r}_h)}}{1 - 2\dot{r}_h}, \qquad S = \pi r_h^2, \tag{2}$$

where $\dot{r}_h = \frac{\partial r_h}{\partial v}$ denotes the event horizon changes with time. Due to this change, it isn't convenient to discuss the Hawking radiation of the Vaidya–Bonner black hole. An effective approach to discuss the Hawking radiation should be in a motion coordinate system. Besides, that the timelike limit surface doesn't coincide with the event horizon also causes some discommodious to the research. Ordering $\Delta = (1 - 2\dot{r}_h)r^2 - 2M(v)r + Q^2(v)$ and introducing the following motion coordinate system

$$R = r - r_h(v), \qquad dR = dr - \dot{r}_h dv, \tag{3}$$

we can get

$$ds^2 = -\Delta r^{-2} dv^2 + 2dvdR + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{4}$$

In which the metrics satisfy Landau's condition of the coordinate clock synchronization, which is helpful to study the Hawking radiation of the black hole. Meanwhile, the event horizon and timelike limit surface are coincident with each other, which imply the geometry optics limit can be relied here. Adopting the WKB approximation [9, 10], we can get the relationship between the tunneling rate and imaginary part of the radiation particle's action as $\Gamma \sim e^{-2\,\mathrm{Im}\,I}$. In the discussion of the Hawking radiation, it is important to derive the action. There are two methods to get it, namely radial geodesic method and Hamilton–Jacobi method. The radial geodesic method was put forwarded by Parikh and Wilczek and extensively applied to the studies of the Hawking radiation of static and stationary black holes. In this method [15–17], in order to get the action, one has to introduce a Painlevé coordinate system and explore the motion equation of the radiation particle. Moreover, massless and massive particles should be differentiated for their motion equations are different. The second method was developed in 1990s and mainly adopted to investigate the non-thermal radiation [1, 14]. The virtue consists in one can get not only the particle's action of the stationary but also that of non-stationary black holes by this method. In this paper, considering the properties of the non-stationary black hole, we use the Hamilton–Jacobi method to obtain the action. Now we turn to the Hamilton–Jacobi method.

The classical action of the radiation particle satisfies relativistic Hamilton–Jacobi equation, namely

$$g^{\mu\nu}(\partial_\mu I + qA_\mu)(\partial_\nu I + qA_\nu) + u^2 = 0. \tag{5}$$

In which u, q, I and $g^{\mu\nu}$ are the mass, charge, action of the radiation particle and the inverse metric tensors derived from the line element (4) respectively. Substituting them into the Hamilton–Jacobi equation yields

$$2(\partial_v I + qA_t)\partial_R I + \Delta r^{-2}(\partial_R I)^2 + r^{-2}(\partial_\theta I)^2 + (r\sin\theta)^{-2}(\partial_\varphi I)^2 + u^2 = 0. \tag{6}$$

Obviously, it is difficult to solve the action I for it is a function of v, R, θ and φ. Considering the properties of the black hole space-time, we carry on the separation of variables as

$$I = W(v, R) + H(\theta) + Y(\varphi) \tag{7}$$

and let $\partial_v I = \partial_v W(v, R) = -\omega$. So (6) becomes

$$\begin{aligned} &-2(\omega - qA_t)\partial_R W(v, R) + \Delta r^{-2}[\partial_R W(v, R)]^2 + r^{-2}[\partial_\theta H(\theta)]^2 \\ &\quad + (r\sin\theta)^{-2}[\partial_\varphi Y(\varphi)]^2 + u^2 = 0, \end{aligned} \tag{8}$$

which is an equation about $W(v, R)$, $H(\theta)$ and $Y(\varphi)$, but our real interest is the expression of $W(v, R)$. Solving it, we have

$$\begin{aligned} W(v, R) &= \int \frac{(\omega - qA_t) + \sqrt{(\omega - qA_t)^2 - \Delta r^{-2}[r^{-2}(\partial_\theta H(\theta))^2 + (r\sin\theta)^{-2}(\partial_\varphi Y(\varphi))^2 + u^2]}}{(1 - 2\dot{r}_h)r^2 - 2M(v)r + Q^2(v)} r^2 dR \\ &= \pi i \frac{r_h^2(\omega - qA_h)}{\sqrt{M^2(v) - Q^2(v)(1 - 2\dot{r}_h)}}, \end{aligned} \tag{9}$$

where $A_h = \frac{Q(v)}{r_h}$ denotes the electromagnetic potential at the event horizon. From (9) and (7), we can get the imaginary part of the action as

$$\mathrm{Im}\, I = \frac{\pi r_h^2(\omega - qA_h)}{\sqrt{M^2(v) - Q^2(v)(1 - 2\dot{r}_h)}}. \tag{10}$$

Clearly, the tunneling rate of the radiation particle can be obtained. However, we find the radiation spectrum only is the leading term, which isn't accordant with recent researches.

This is because the self-gravitational interaction and unfixed background space-time weren't taken into account. Now let's considered these and move on discussing the Hawking radiation of the black hole. In this paper, note the particle with energy ω and charge q tunnels across the event horizon in from of S-wave. When the particle tunnels out, the energy and charge of the black hole will become as $M(v)-\omega$ and $Q(v)-q$.

In this paper, we consider the following case, namely the tunneling radiation in a short time Δv (Δv expresses the time that a particle tunnels across the potential barrier). Since that the particle tunnels across the barrier is an instantaneous process, the time Δv is infinity short. Therefore we can suppose it's too late for other particles to be absorbed or evaporated by the black hole in this time. Although the black hole is non-stationary and the mass and charge et al. parameters change with time, these parameters should be certain in the time Δv. In this time, and then we can study the tunneling radiation of the non-stationary Vaidya–Bonner black hole following the case of stationary black holes. Taking the self-gravitational interaction into account, we can obtain the imaginary part of actual action as

$$\begin{aligned} \operatorname{Im} I &= \pi \int_{(0,0)}^{(\omega,q)} \frac{r_h'^2 (d\omega' - A_h' dq')}{\sqrt{(M(v)-\omega')^2 - (Q(v)-q')^2(1-2\dot{r}_h')}} \\ &= -\pi r_h' \int_{(M(v),Q(v))}^{(M(v)-\omega,Q(v)-q)} \frac{r_h' dM' - Q' dQ'}{\sqrt{M'^2 - Q'^2(1-2\dot{r}_h')}}, \end{aligned} \tag{11}$$

where

$$\begin{aligned} &M' = M(v)-\omega', \qquad Q' = Q(v)-q', \qquad A_h' = \frac{Q(v)-q'}{r_h'}, \qquad \dot{r}_h' = \frac{\partial r_h'}{\partial v}, \\ &r_h' = \frac{[M(v)-\omega'] + \sqrt{[M(v)-\omega']^2 - [Q(v)-q']^2(1-2\dot{r}_h')}}{1-2\dot{r}_h'}. \end{aligned} \tag{12}$$

Since $\dot{r}_h$ is the function of $M(v)$, $Q(v)$ and r_h, it is very difficult to get the result from (11) by direct integral. From the entropy of the black hole, $S = \pi r_h^2$, we have

$$\begin{aligned} dS = 2\pi r_h \Bigg[&\frac{r_h dM - Q dQ}{\sqrt{M^2 - Q^2(1-2\dot{r}_h)}} \\ &+ \left(\frac{Q^2}{(1-2\dot{r}_h)\sqrt{M^2 - Q^2(1-2\dot{r}_h)}} + \frac{2r_h}{1-2\dot{r}_h} \right) \left(\frac{\partial \dot{r}_h}{\partial M} dM + \frac{\partial \dot{r}_h}{\partial Q} dQ \right) \Bigg]. \end{aligned} \tag{13}$$

Comparing (11) with (13), there is

$$\begin{aligned} \operatorname{Im} I = &-\frac{\Delta S_{BH}}{2} \\ &+ \pi r_h' \int_{(M(v),Q(v))}^{(M(v)-\omega,Q(v)-q)} \left(\frac{Q'^2}{(1-2\dot{r}_h')\sqrt{M'^2 - Q'^2(1-2\dot{r}_h')}} + \frac{2r_h'}{1-2\dot{r}_h'} \right) \\ &\times \left(\frac{\partial \dot{r}_h'}{\partial M'} dM' + \frac{\partial \dot{r}_h'}{\partial Q'} dQ' \right), \end{aligned} \tag{14}$$

where $\Delta S_{BH} = S_{BH}[M(v)-\omega, Q(v)-q] - S_{BH}[M(v), Q(v)]$ is the change of Bekenstein–Hawking entropy. So the tunneling rate is

$$\begin{aligned}\Gamma &\sim \mathrm{e}^{-2\operatorname{Im} I}\\ &= \exp\left[\Delta S_{BH} - 2\pi r'_h \int_{(M(v),Q(v))}^{(M(v)-\omega,Q(v)-q)} \left(\frac{Q'^2}{(1-2\dot{r}'_h)\sqrt{M'^2-Q'^2(1-2\dot{r}'_h)}} + \frac{2r'_h}{1-2\dot{r}'_h}\right)\right.\\ &\qquad \left.\times \left(\frac{\partial \dot{r}'_h}{\partial M'} dM' + \frac{\partial \dot{r}'_h}{\partial Q'} dQ'\right)\right]. \end{aligned} \tag{15}$$

Obviously, the radiation spectrum deviates from the purely thermal one and the tunneling rate is related to two parts: one part is the change of Bekenstein–Hawking entropy; another part is related to the integral to the black hole mass and charge. Our result doesn't satisfy the unitary theory and is different from the Parikh and Wilczek's result.

In summary, considering the unfixed background space-time and self-gravitational interaction, we have discussed the Hawking radiation of the Vaidya–Bonner black hole by Hamilton–Jacobi method. The result isn't in accordance with Parikh and Wilczek's result and implies the information loss is possible, which support the view of information loss [7, 22].

Acknowledgements The work is supported by National Nature Science Foundation of China (grant No. 10347008) and Sichuan Province Science and Technology Department (grant No. 05JY029-092).

References

1. Angheben, M., Nadalini, M., Vanzo, L., Zerbini, S.: JHEP **05**, 014 (2005)
2. Bonner, W.B., Vaidya, P.C.: Gen. Relativ. Gravit. **1**, 127 (1970)
3. Chen, D.Y., Jiang, Q.Q., Li, H.L., Yang, S.Z.: Chin. Phys. **15**, 1425 (2006)
4. Damour, T., Ruffini, R.: Phys. Rev. D **20**, 239 (1976)
5. Hawking, S.W.: Nature **30**, 248 (1974)
6. Hawking, S.W.: Commun. Math. Phys. **43**, 199 (1975)
7. Hawking, S.W.: Phys. Rev. D **72**, 084013 (2005)
8. Hemming, S., Keski-Vakkuri, E.: Phys. Rev. D **64**, 044006 (2001)
9. Kraus, P., Keski-Vakkuri, E.: Nucl. Phys. B **491**, 219 (1997)
10. Kraus, P., Parentani, R.: Nucl. Phys. B **575**, 333 (2000)
11. Li, H.L., Yang, S.Z., Jiang, Q.Q., Qi, D.J.: Phys. Lett. B **641**, 139 (2006)
12. Liu, L., Xu, D.: Chin. Phys. **1**, 934 (1981)
13. Medved, A.J.M.: Phys. Rev. D **66**, 124009 (2002)
14. Medved, A.J.M., Vagenas, E.C.: Mod. Phys. Lett. A **20**, 2449 (2005)
15. Parikh, M.K., Wilczek, F.: Phys. Rev. Lett. **85**, 5042 (2000)
16. Parikh, M.K.: Phys. Lett. B **546**, 189 (2002)
17. Parikh, M.K.: Int. J. Mod. Phys. D **13**, 2351 (2004)
18. Sannan, S.: Gen. Relativ. Gravit. **20**, 239 (1988)
19. Yang, S.Z.: Int. J. Theor. Phys. **45**, 965 (2006)
20. Yang, S.Z., Li, H.L., Jiang, Q.Q., Liu, M.Q.: Sci. China (2007, in press)
21. Zhao, Z., Gui, Y.X., Liu, L.: Acta. Astrophys. Sin. **1**, 141 (1981) (in Chinese)
22. Zhang, J.Y., Hu, Y.P., Zhao, Z.: Mod. Phys. Lett. A **21**, 1865 (2006)
23. Zhang, J.Y., Zhao, Z.: Mod. Phys. Lett. A **20**, 1673 (2005)
24. Zhang, J.Y., Zhao, Z.: Phys. Lett. B **618**, 14 (2005)
25. Zhang, J.Y., Zhao, Z.: JHEP **0505**, 10055 (2005)
26. Zhang, J.Y., Zhao, Z.: Phys. Lett. B **638**, 110 (2006)